UNIT

5 Polynomials

What You'll Learn

- Recognize, write, describe, and classify polynomials.
- Represent polynomials using tiles, pictures, and algebraic expressions.
- Add and subtract polynomials.
- Multiply and divide a polynomial by a monomial.

Why It's Important

Polynomials are used by

- homeowners to calculate mortgage and car payments
- computer technicians to encode information, such as PIN numbers for ATM machines and debit cards

Key Words

term
variable term
constant term
variable
coefficient of the variable
polynomial
degree of a polynomial

monomial
binomial
trinomial
simplify a polynomial
like terms
unlike terms
distributive property

5.1 Skill Builder

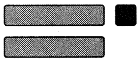
Modelling Expressions

We can use algebra tiles to model an expression.
One ▨ represents +1. One ■ represents −1.

One ▭ represents any variable, such as x or n.

One ▬ represents $-x$ or $-n$.

There are 2 ▭.
They represent $2x$.



There is 1 ■.
It represents −1.

So, the tiles represent the expression $2x - 1$.

There are 3 ▬.
They represent $-3a$.

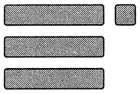
There are 2 ▨.
They represent +2.

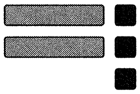
So, the tiles represent the expression $-3a + 2$.

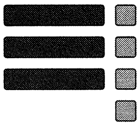
We can use any letter as the variable.

Check

1. Which expression does each set of tiles represent?

a)  ____________

b)  ____________

c)  ____________

d)  ____________

2. Sketch algebra tiles to model each expression.

a) $s + 4$

b) $5b - 3$

c) $-4n + 5$

d) $-6w - 1$

5.1 Modelling Polynomials

FOCUS **Model, write, and classify polynomials.**

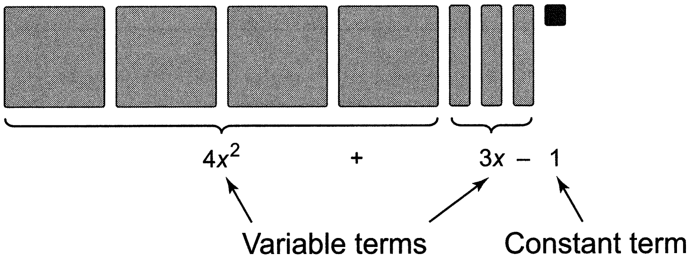
Some expressions contain x^2 terms.

We use ▨ to represent x^2.

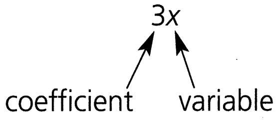
When the variable is n, the tile is called the n^2-tile.

We use ■ to represent $-x^2$.

For the expression $4x^2 + 3x - 1$:



In the term $3x$, the **variable** is x and the **coefficient of the variable** is 3.



An algebraic expression like this one is also called a **polynomial**.

Example 1 Modelling Polynomials with Algebra Tiles

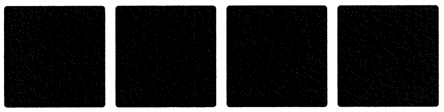
Use algebra tiles to model each polynomial.

a) $-4t^2$

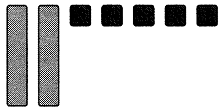
b) $2n - 5$

Solution

a) To represent $-4t^2$, use 4 ■.



b) To represent $2n$, use 2 ▬.
To represent -5, use 5 ■.



Check

1. Sketch algebra tiles to model each polynomial.

a) -3

b) $2x + 3$

c) $2e^2 - e + 2$

d) $-3d^2 + 2d - 5$

Example 2 Recognizing the Same Polynomials in Different Variables

Which of these polynomials can be represented by the same algebra tiles?

a) $2x^2 + 7x - 4$

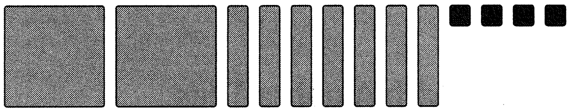
b) $-4 + 2b^2 - 7b$

c) $7s - 4 + 2s^2$

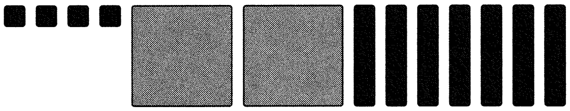
Solution

Select the tiles that match each term.

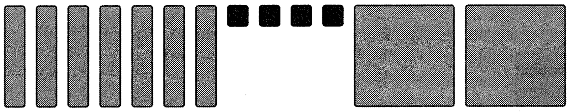
a) $2x^2 + 7x - 4$



b) $-4 + 2b^2 - 7b$



c) $7s - 4 + 2s^2$



The variable used to name a tile does not matter.
In parts a and c, the same algebra tiles are used.
Since $2x^2 + 7x - 4$ and $7s - 4 + 2s^2$ can be represented
by the same tiles, the expressions represent the same polynomial.

The order in which the terms are written does not matter.

Check

1. Which of these polynomials can be represented by the same algebra tiles?

a) $3s^2 - 2s + 5$

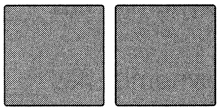
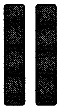
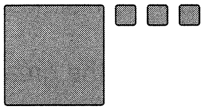
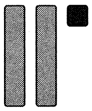
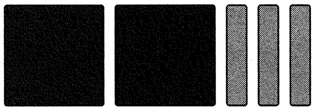
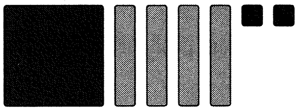
b) $5 - 3a^2 - 2a$

c) $-2c + 5 - 3c^2$

The same tiles are used in parts _____ and _____.

So, ___________________ and ___________________ represent the same polynomial.

There are different **types** of polynomials, depending on the number of terms.
The **degree of a polynomial** tells you the greatest exponent of any term.

Type	Number of Terms	Example	Model	Degree
Monomial	1	$2s^2$		2
		$-2n$		1
		4		2
Binomial	2	$x^2 + 3$		2
		$2a - 1$		1
		$-2b^2 + 3b$		2
Trinomial	3	$-c^2 + 4c - 2$		2

A monomial has 1 type of tile.

A constant term has degree 0.

A binomial has 2 different types of tiles.

A trinomial has 3 different types of tiles.

An algebraic expression that contains a term with a variable in the denominator, such as $\frac{5}{n}$, or the square root of a variable, such as $\sqrt{n}$, is not a polynomial.

Practice

1. Sketch algebra tiles to model each polynomial.

a) $a^2 + 6$

____

b) $y^2 - y + 3$

____

c) $-2m^2 + 3m - 4$

____

d) $2x^2 + 5x + 4$

____

2. Is the polynomial a monomial, binomial, or trinomial?

a) $-7t$ The polynomial has ___ term, so it is a ____________________.

b) $8d^2 + 7$ The polynomial has ___ terms, so it is a ____________________.

c) $s^2 + 5s - 6$ The polynomial has ___ terms, so it is a ____________________.

d) $4t - 12$ The polynomial has ___ terms, so it is a ____________________.

e) -15 The polynomial has ___ term, so it is a ____________________.

3. Name the degree of each polynomial.

a) $5a^2 - 3a + 6$ The term with the greatest exponent is $5a^2$.
It has exponent _____.
So, the polynomial has degree _____.

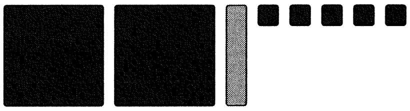
b) $4b - 6$ The term with the greatest exponent is _____.
It has exponent _____.
So, the polynomial has degree _____.

c) $4d^2 - 3d$ The term with the greatest exponent is _____.
It has exponent _____.
So, the polynomial has degree _____.

d) -4 -4 can be written as $-4x$——.
So, the polynomial has degree _____.

4. Write the polynomial represented by each set of tiles.

a) Use the variable *f*.



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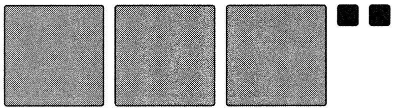
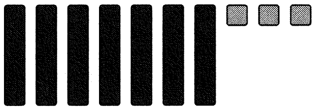
b) Use the variable *n*.



____

c) Use the variable *p*.



____

5. Choose a set of tiles from question 4.
Write another polynomial that can be represented by the same set of tiles.

____

____

6. Identify the polynomials that can be represented by the same set of algebra tiles.

a) $x^2 + 3x - 1$ 1 ▨, ____ ▭, and ____ ■

b) $4r^2 - 5r + 9$ ____________________________

c) $9 + 4z^2 - 5z$ ____________________________

d) $3s + 1 + s^2$ ____________________________

Parts ____ and ____ use the same algebra tiles.
So, __________________ and __________________ both represent
the same polynomial.

5.2 Skill Builder

Modelling Integers

One ▨ represents +1.
One ■ represents –1.
One ▨ and one ■ combine to model 0.

▨ } +1
■ } –1
We call this a **zero pair.**

We can model any integer in many ways.

Each set of tiles below models +3.

▨ ▨ ▨

▨ ▨ ▨ ▨
■

▨ ▨ ▨ ▨ ▨
■ ■

Each pair of 1 ▨ and 1 ■ makes a zero pair.

Check

1. Write the integer modelled by each set of tiles.

a) ▨ ▨ ▨ ▨ ▨
■ ■ ■

____

b) ▨ ▨
■ ■ ■

____

c) ▨
■ ■ ■ ■

____

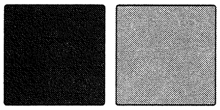
d) ▨ ▨ ▨
■ ■ ■

____

5.2 Like Terms and Unlike Terms

FOCUS **Simplify polynomials by combining like terms.**

These are all zero pairs:



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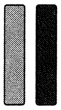

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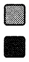
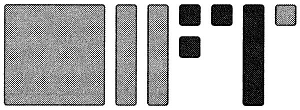


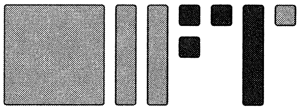
We can use zero pairs to simplify algebraic expressions.

Example 1 Combining Like Tiles and Removing Zero Pairs

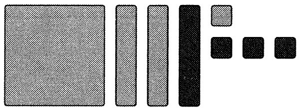
Simplify this tile model.
Write the polynomial that the remaining tiles represent.



Solution

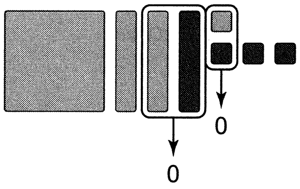


Group like tiles.

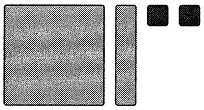


Matching tiles have the same size and shape.

Remove zero pairs.



The tiles that remain are:



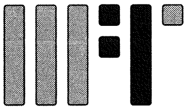
They represent: $x^2 + x - 2$

When there is only 1 of a type of tile, we omit the coefficient 1.

Check

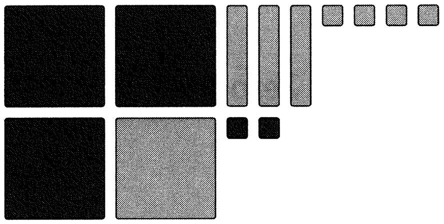
1. Simplify each tile model.
Write the polynomial that the remaining tiles represent.

a)



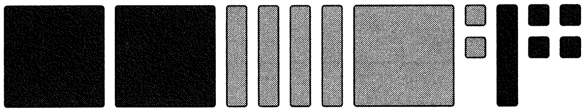
Remaining tiles: ____________________ Polynomial: ________________

b)



Remaining tiles: ____________________ Polynomial: ________________

c)



Remaining tiles: ____________________ Polynomial: ________________

Terms that can be represented by matching tiles are called **like terms**.

Like terms:	x^2 and $-2x^2$	$4s$ and $-s$	6 and -2	$5w$ and w
Unlike terms:	$3s$ and s^2	$2x$ and -5	$3d^2$ and 7	

We can **simplify a polynomial** by adding the coefficients of like terms.
To simplify $-5x + 2x$, add the integers: $-5 + 2 = -3$
So, $-5x + 2x = -3x$

Example 2 Simplifying a Polynomial Symbolically

Simplify:

a) $3a + 6 + a - 4$ **b)** $-x^2 + 4x - 5 + 3x^2 - 4x + 1$

Solution

a) $3a + 6 + a - 4$ Group like terms.
$= 3a + 1a + 6 - 4$ Add the coefficients of like terms.
$= 4a + 2$

b) $-x^2 + 4x - 5 + 3x^2 - 4x + 1$ Group like terms.
$= -x^2 + 3x^2 + 4x - 4x - 5 + 1$ Add the coefficients of like terms.
$= 2x^2 + 0x - 4$
$= 2x^2 - 4$

We omit a term when its coefficient is 0.

Check

1. Simplify each polynomial.

a) $5d + 2 + 3d - 1$ Group like terms.
$= 5d + 3d + 2 - 1$ Add the coefficients of like terms:
= ____d + ____ 5 + 3 = ____ and 2 + (−1) = ____

b) $2a^2 - 3a + 5a^2 + 7a$ Group like terms.
= ________________ Add the coefficients of like terms:
= ________________ ____ + ____ = ____ and ____ + ____ = ____

c) $-x^2 + 4x - 5 + 2x^2 + x + 3$
= ________________
____
____

We omit the coefficient when it is 1.

d) $2x^2 + 6x + 7 - 2x^2 + 7x - 11$
= ________________
____
____

Practice

1. What is the coefficient of each term?

a) $2x^2$ _____ **b)** $6w$ _____ **c)** $-3x$ _____

d) $7t$ _____ **e)** b _____ **f)** $-s$ _____

2. a) Which of these terms are like $3z^2$?

$5z$ $-z^2$ -9 $-6z$ $2z^2$ -11 $-4z^2$

$3z^2$ has variable _____ and exponent _____.

Find all terms with the same variable and exponent: _______________

b) Which of these terms are like $-5x$?

$-4x$ $-3x^2$ -2 $7x$ $5x^2$ 8 $-x$ $-5t$

$-5x$ has variable _____ and exponent _____.

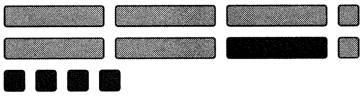
Find all terms with the same variable and exponent: _______________

3. Simplify each tile model.
Write the polynomial that the remaining tiles represent.

a)

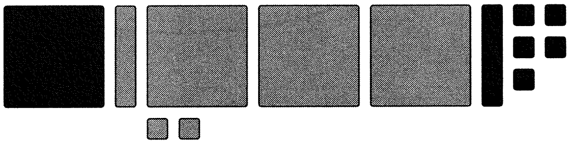
Remaining tiles: ___________________ Polynomial: ___________________

b)



Remaining tiles: ___________________ Polynomial: ___________________

c)



Remaining tiles: ___________________ Polynomial: ___________________

4. Add integers to combine like terms.

a) $-3c + 5c$ $-3 + 5 =$ _____

$-3c + 5c =$ _____

b) $4s - s$ $4 + (-1) =$ _____

$4s - s =$ _____

c) $-2x^2 + 7x^2$ _____ + _____ = _____

____

d) $8e^2 - 8e^2$ __________________

____

5. Simplify each polynomial.

a) $5m + 7 - 2m + 1$

= __________________ Group like terms.

= __________________ Add the coefficients of like terms.

b) $7c^2 - 6c - 4c^2 + c$

= __________________ Group like terms.

= __________________ Add the coefficients of like terms.

c) $11 - 9v + v^2 + 2 - v$

= __________________

= __________________

____

d) $-7f^2 + 12f - 2 -3f^2 - 3f + 5$

= __________________

= __________________

We usually write a polynomial so the exponents of the variable decrease from left to right.

A polynomial in simplified form is equal to the original polynomial.

6. Identify and explain any errors you find.

a) $3x + 2 = 5x$ __________________

____

b) $5s + 3s = 8s^2$ __________________

____

____

____

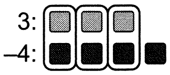
c) $x^2 - x^2 = 0$ __________________

____

5.3 Skill Builder

Adding Integers

To add two integers: 3 + (−4)
We can model each integer with tiles.
Circle zero pairs.



There are 3 zero pairs.
There is 1 tile left.
It models −1.
So, 3 + (−4) = −1 ← *This is an addition sentence.*

Check

1. Sketch tiles to show the sum of each pair of integers.
Write an addition sentence each time.

a) 4:
5: ____________________

b) 6:
−2: ____________________

c) −3:
−5: ____________________

d) 3:
−3: ____________________

e) 5:
−8: ____________________

5.3 Adding Polynomials

FOCUS Use different strategies to add polynomials.

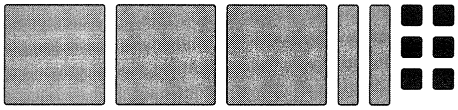
Example 1 Adding Polynomials with Algebra Tiles

Use algebra tiles to model $(3s^2 + 2s - 6) + (-s^2 - 2s + 1)$.
Write an addition sentence.

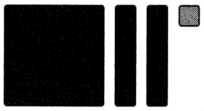
Solution

Model each polynomial.

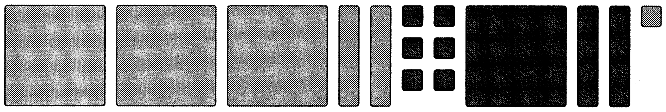
$3s^2 + 2s - 6$



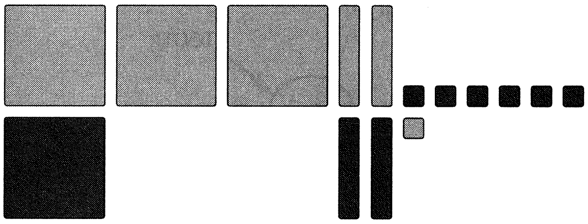
$-s^2 - 2s + 1$



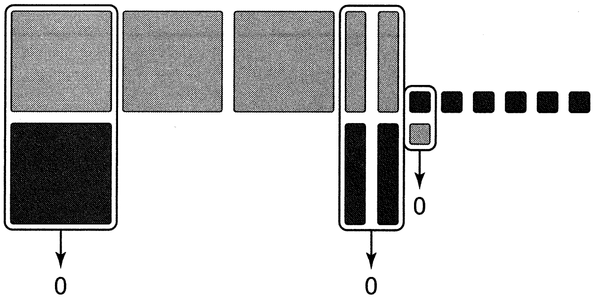
Combine the tiles.



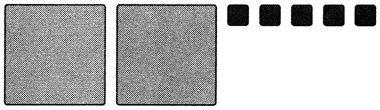
Group matching tiles.



Remove zero pairs.



The remaining tiles are:



They represent: $2s^2 - 5$

The addition sentence is: $(3s^2 + 2s - 6) + (-s^2 - 2s + 1) = 2s^2 - 5$

Check

1. Sketch algebra tiles to model each sum.
Then write the sum.

a) $(6p + 4) + (-2p + 1)$

Remaining tiles: ____________________
So, $(6p + 4) + (-2p + 1) =$ ____________

b) $(2x^2 - x + 1) + (x^2 - 3)$

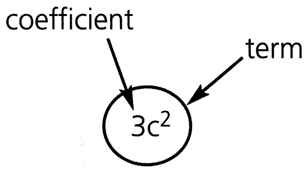
Remaining tiles: ____________________
So, $(2x^2 - x + 1) + (x^2 - 3) =$ ____________

c) $(3e^2 + 6e - 5) + (-4e^2 - 3e + 8)$

Remaining tiles: ____________________
So, $(3e^2 + 6e - 5) + (-4e^2 - 3e + 8) =$ ____________

Algebra tiles are not always available.
To add polynomials without tiles:

- remove the brackets
- add the coefficients of like terms



Example 2 Adding Polynomials Symbolically

Add: $(3c^2 + 5c - 6) + (2c^2 - 3c + 4)$

Solution

$(3c^2 + 5c - 6) + (2c^2 - 3c + 4)$ Remove the brackets.

$= 3c^2 + 5c - 6 + 2c^2 - 3c + 4$ Group like terms.

$3c^2$ and $2c^2$ are like terms.

$= \underbrace{3c^2 + 2c^2} + \underbrace{5c - 3c} - \underbrace{6 + 4}$ Add the coefficients of like terms.

$= \quad 5c^2 \quad + \quad 2c \quad -2$

Check

1. Add.

a) $(7g - 8) + (3g + 1)$	Remove the brackets.
$= 7g - 8 + 3g + 1$	Group like terms.
$= \underbrace{7g + 3g} - \underbrace{8 + 1}$	Add the coefficients of like terms.
= ________________	7 + 3 = _____ and −8 + 1 = _____
b) $(2a^2 - 9a) + (-5a^2 + 12a)$	Remove the brackets.
= ________________	Group like terms.
= ________________	Add the coefficients of like terms.
= ________________	_____ + _____ = _____ and _____ + _____ = _____
c) $(-c^2 + 11c - 3) + (4c^2 + 5)$	***Recall:*** $-c^2$ ***has coefficient*** -1***.***
= ________________	
= ________________	
= ________________	

We can also add 2 polynomials by aligning like terms vertically.

Example 3 Adding Polynomials Vertically

Add: $(2m + 9) + (3m^2 + m - 14)$

Solution

To add the polynomials, remove the brackets and align like terms vertically.

In $3m^2 + m - 14$, the term m has coefficient 1, so write m as $1m$.

$$\begin{array}{r} 2m + \ \ 9 \\ +\ 3m^2 + 1m - 14 \\ \hline 3m^2 + 3m - \ \ 5 \end{array}$$

Add the coefficients of like terms.

$$\begin{array}{ccc} 0 & 2 & 9 \\ \underline{+3} & \underline{+1} & \underline{+(-14)} \\ 3 & 3 & -5 \end{array}$$

So, $(2m + 9) + (3m^2 + m - 14) = 3m^2 + 3m - 5$

Check

1. Add vertically.

a) $(2x + 3) + (4x + 8)$

$2x + 3$

$\underline{+\ 4x + 8}$

$\_\_\_\_x + \_\_\_\_$

b) $(5p^2 + 12) + (-2p^2 + 3p - 7)$

$5p^2 \qquad\quad + 12$

$\underline{+\ -2p^2 \ + \ 3p \ - \ 7}$

____

c) $(-6b^2 - 2b + 8) + (9b - b^2 - 19)$

____

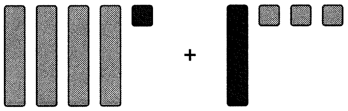
____

____

Practice

1. Write the addition sentence modelled by each set of tiles.
Use the variable x.

a)



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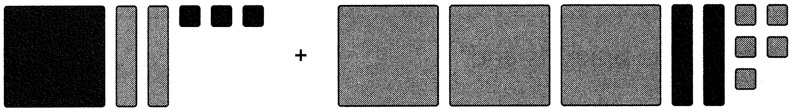
b)



____

2. Sketch algebra tiles to model each sum.
Then write the sum.

a) $(-5w + 8) + (7w - 3) =$ ____________

Remaining tiles: ________________________

b) $(-6t^2 - 3t + 2) + (4t^2 - t + 1) =$ ____________________

Remaining tiles: ____________________________________

3. Add horizontally.

a) $(2r - 3) + (3r - 1)$

$= 2r - 3 + 3r - 1$

$= 2r + 3r - 3 - 1$

$=$ ____$r -$ ____

Remove the brackets.

Group like terms.

Add the coefficients of like terms.

$2 + 3 =$ ____ and $-3 + (-1) =$ ____

b) $(7h^2 - 2h) + (-4h^2 + 9h - 4)$

= ____________________

= ____________________

= ____________________

c) $(-2y^2 + 6y - 1) + (2y^2 - 6y + 5)$

= ____________________

= ____________________

= ____________________

4. Add vertically.

a) $(9r + 7) + (2r - 3)$

$$\begin{array}{r} 9r + 7 \\ + \; 2r - 3 \\ \hline \end{array}$$

____$r +$____

b) $(-a^2 + 4a) + (-3a^2 + 2a - 5)$

$$\begin{array}{r} -1a^2 + 4a \\ + \; -3a^2 + 2a - 5 \\ \hline \end{array}$$

____

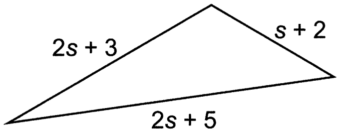
c) $(8v - 2v^2 - 3) + (9 + 6v^2 - 10v)$

____

____

____

5. Find the perimeter of this triangle.



***Perimeter** is the distance around a shape. To find the perimeter, add the side lengths.*

Perimeter = ____________________

= ____________________

= ____________________

= ____________________

Remove the brackets.

Group like terms.

Add coefficients of like terms.

5.4 Skill Builder

Subtracting Integers Symbolically

To subtract an integer without tiles, we add the opposite integer.
−3 and 3, −6 and 6, and −15 and 15 are opposite integers.

We omit the + sign when the integer is positive.

To subtract: (−4) − (−3)
Add the opposite integer.
The opposite of −3 is 3.
And, (−4) + 3 = −1
So, (−4) − (−3) = −1
We can use algebra tiles to check:
Model −4:
■ ■ ■ ■
Take away −3:
$\blacksquare \boxed{\blacksquare\,\blacksquare\,\blacksquare} \rightarrow -3$
One ■ remains.
So, (−4) − (−3) = −1

Check

1. Subtract.

a) 6 − (−2):

The opposite of −2 is ____.
Add the opposite: 6 + ____ = ____
So, 6 − (−2) = ____

b) 3 − (4):

The opposite of 4 is ____.
Add the opposite: ____________________
So, 3 − (4) = ____

c) (−8) − (−5):

The opposite of −5 is ____.
Add the opposite: ____________________
So, ____________________

d) (−9) − (4):

The opposite of 4 is ____.
Add the opposite: ____________________
So, ____________________

5.4 Subtracting Polynomials

FOCUS Use different strategies to subtract polynomials.

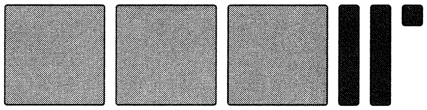
To subtract a polynomial, we subtract each term of the polynomial.

Example 1 Subtracting Polynomials with Algebra Tiles

Use algebra tiles to model $(3b^2 - 2b - 1) - (-2b^2 - b + 2)$.
Write a subtraction sentence.

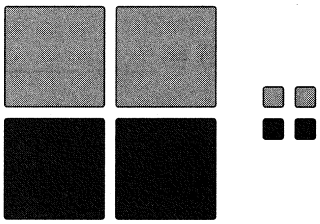
Solution

Model: $3b^2 - 2b - 1$

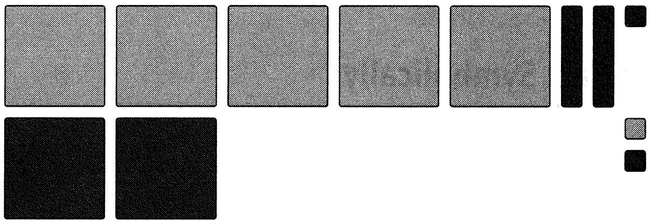


To subtract $-2b^2 - b + 2$, take away 2 ■, 1 ▬, and 2 ▨.

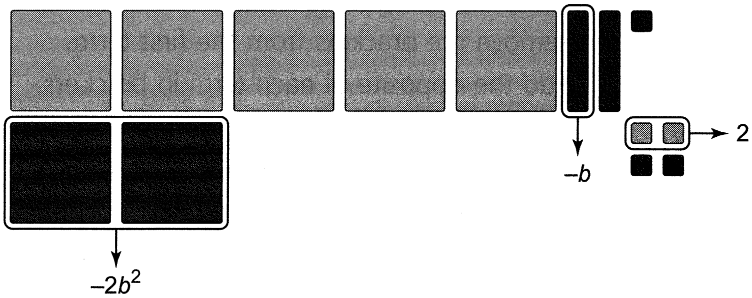
There are no ■ or ▨ to take away.
So, add 2 zero pairs of each tile:



So, these tiles also model $3b^2 - 2b - 1$.



Take away the tiles for $-2b^2 - b + 2$.



The remaining tiles represent: $5b^2 - b - 3$
The subtraction sentence is: $(3b^2 - 2b - 1) - (-2b^2 - b + 2) = 5b^2 - b - 3$

Check

1. Use algebra tiles to model each difference.
Sketch the tiles that remain, then write the difference.

a) $(4p + 3) - (2p + 1)$

Remaining tiles: ____________________________

So, $(4p + 3) - (2p + 1) =$ ______________

b) $(5t + 1) - (-2t + 3)$

Remember to add zero pairs when there are not enough tiles to subtract.

Remaining tiles: ____________________________

So, $(5t + 1) - (-2t + 3) =$ ______________

c) $(3e^2 + 2e - 4) - (4e^2 + 3e - 2)$

Remaining tiles: ____________________________

So, $(3e^2 + 2e - 4) - (4e^2 + 3e - 2) =$ ______________

To subtract integers without tiles, we can add the opposite integer.
To subtract polynomials without tiles, we can add the opposite terms.

Example 2 Subtracting Polynomials Symbolically

Subtract: $(-5k^2 + 2k - 6) - (3k^2 - 4k + 1)$

Solution

$(-5k^2 + 2k - 6) - (3k^2 - 4k + 1)$	Remove the brackets from the first term.
$= -5k^2 + 2k - 6 - (3k^2 - 4k + 1)$	Add the opposite of each term in brackets.
$= -5k^2 + 2k - 6 + (-3k^2 + 4k - 1)$	Remove the brackets.
$= -5k^2 + 2k - 6 - 3k^2 + 4k - 1$	Group like terms.
$= -5k^2 - 3k^2 + 2k + 4k - 6 - 1$	Add the coefficients of like terms.
$= -8k^2 + 6k - 7$	

Check

1. Subtract.

a) $(8f - 3) - (7f + 5)$ Remove the brackets from the first term.

= ____________ $- (7f + 5)$ The opposite of $7f$ is: ________

The opposite of 5 is: ________

Add the opposites.

$= 8f - 3 +$ ________________ Remove the brackets.

= ________________________ Group like terms.

= ________________________ Add the coefficients of like terms.

= ________________________

b) $(2 + 5g - 7g^2) - (9g - 4g^2 + 2)$

= ____________________________

= ____________________________

= ____________________________

= ____________________________

= ____________________________

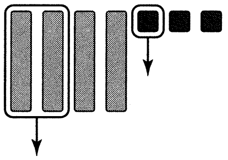
= ____________________________

Remember to write the polynomial in descending order.

Practice

1. Write the subtraction sentence modelled by each set of tiles.

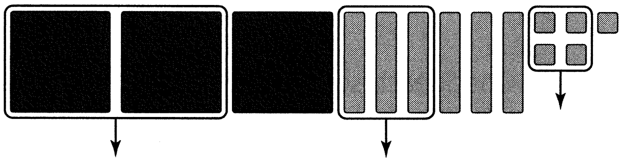
a)



____

____

b)



____

____

2. Use algebra tiles to model each difference.
Sketch the tiles that remain, then write the difference.

a) $(3r + 2) - (-2r + 3)$

Remaining tiles: ________________

So, $(3r + 2) - (-2r + 3) =$ ____________

b) $(-4v^2 + 5v - 1) - (-3v^2 + 4v - 2)$

Remaining tiles: ________________

So, $(-4v^2 + 5v - 1) - (-3v^2 + 4v - 2)$

= ________________

3. Write the opposite of each term.

a) -9: ____ **b)** $3r$: ____ **c)** $-2s^2$: ____ **d)** t: ____

4. Subtract.

a) $(4p + 1) - (p + 10)$	Remove the brackets from the first term.
$=$ __________ $- (p + 10)$	The opposite of p is: _____
	The opposite of 10 is: _____
	Add the opposites.
$= 4p + 1 +$ ____________________	Remove the brackets.
$=$ ______________________________	Group like terms.
$=$ ______________________________	Add the coefficients of like terms.
$=$ ______________________________	

b) $(3h^2 + 5h - 4) - (h^2 - 4h + 6)$	Remove the brackets from the first term.
$=$ ______________________________	Add the opposites.
$=$ ______________________________	Remove the brackets.
$=$ ______________________________	Group like terms.
$=$ ______________________________	Add the coefficients of like terms.
$=$ ______________________________	

c) $(4q^2 + 3) - (3q - q^2 + 3)$

$=$ ______________________________

$=$ ______________________________

$=$ ______________________________

$=$ ______________________________

$=$ ______________________________

5. Check each solution. Identify any errors and correct them.

a) $(7x^2 + 3x + 7) - (3x^2 - 4)$	$(7x^2 + 3x + 7) - (3x^2 - 4)$
$= 7x^2 + 3x + 7 - 3x^2 - 4$	$=$ ________________________
$= 7x^2 - 3x^2 + 3x + 7 - 4$	$=$ ________________________
$= 4x^2 + 3x + 3$	$=$ ________________________

b) $(3a^2 - 2a + 4) - (2a^2 + 3)$	$(3a^2 - 2a + 4) - (2a^2 + 3)$
$= 3a^2 - 2a + 4 - 2a^2 - 3$	$=$ ________________________
$= 3a^2 - 2a^2 - 2a + 4 - 3$	$=$ ________________________
$= a^2 + 2a - 3$	$=$ ________________________

CHECKPOINT

Can you ...

- Recognize, write, describe, and classify polynomials?
- Represent polynomials using tiles, pictures, and algebraic expressions?
- Simplify polynomials by combining like terms?
- Add and subtract polynomials?

5.1 **1.** Is the polynomial a monomial, binomial, or trinomial?

a) -9 The polynomial has ____ term, so it is a ____________.

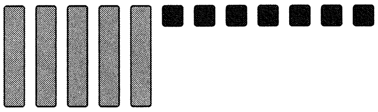
b) $3f - 5$ The polynomial has ____ terms, so it is a ____________.

c) $2s^2 - s + 1$ The polynomial has ____ terms, so it is a ____________.

d) $-a^2 + 2a$ The polynomial has ____ terms, so it is a ____________.

2. Write the polynomial represented by each set of tiles.

a) Use the variable *g*.



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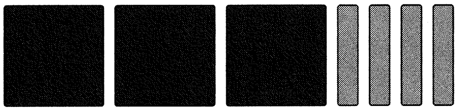
b) Use the variable *r*.



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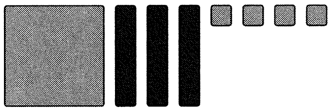
c) Use the variable *w*.



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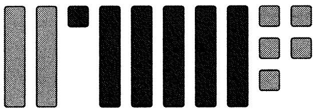
5.2 **3.** Simplify each tile model.
Write the polynomial that the remaining tiles represent.

a)



Remaining tiles: ________________ Polynomial: ________________

b)

Remaining tiles: ____________ Polynomial: ____________

4. Simplify each polynomial.

a) $8e - 9 - 5e + 4$ Group like terms.
$=$ ____________ Add the coefficients of like terms.
$=$ ____________

b) $4d^2 - 3d + 11 - d^2 + 5d - 13$
$=$ ____________________
$=$ ____________________

5.3 **5.** Sketch tiles to model each sum.
Then write the sum.

a) $(4v - 4) + (-2v + 7)$

Remaining tiles: ______________
So, $(4v - 4) + (-2v + 7) =$ ________

b) $(6u^2 - 5u - 7) + (-3u^2 + 3u + 7)$

Remaining tiles: ____________________
So, $(6u^2 - 5u - 7) + (-3u^2 + 3u + 7) =$ ________

6. Add.

a) $(3t + 11) + (-7t - 4)$

$$\begin{array}{r} 3t + 11 \\ + -7t - \ \ 4 \\ \hline \_\_t + \_\_ \end{array}$$

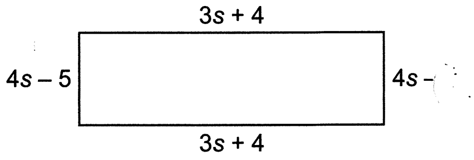
b) $(10y^2 - 9) + (-3y^2 + 4y - 2)$

$$\begin{array}{rrr} 10y^2 & & -\ \ 9 \\ + -3y^2 & + \ 4y & -\ \ 2 \\ \hline & & \end{array}$$

____

7. Find the perimeter of this rectangle.

Perimeter $=$ ______________________
$=$ ______________________
$=$ ______________________
$=$ ______________________



5.4 **8.** Use algebra tiles to model each difference.
Sketch the tiles that remain, then write the difference.

a) $(5n - 6) - (-n - 3)$

Remaining tiles: ______________________
So, $(5n - 6) - (-n - 3) =$ _________

b) $(-v^2 + 3v - 5) - (-v^2 + 4v + 2)$

Remaining tiles: ______________________
So, $(-v^2 + 3v - 5) - (-v^2 + 4v + 2) =$ _________

9. Subtract.

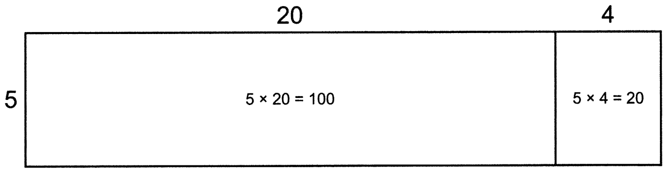
a) $(11h + 3) - (9h - 2)$ Remove the brackets from the first term.
= ________ $- (9h - 2)$ Add the opposites.
= ________ + (________) Remove the brackets.
= ____________________________ Group like terms.
= ____________________________ Add the coefficients of like terms.
= ____________________________

b) $(7j^2 - 11j - 7) - (12j^2 - 8j - 3)$
= ________________________________
= ________________________________
= ________________________________
= ________________________________
= ________________________________

5.5 Skill Builder

The Distributive Property

We can use this diagram to model 5 × 24.



This diagram shows:

$$\begin{aligned} 5 \times 24 &= 5 \times (20 + 4) \\ &= (5 \times 20) + (5 \times 4) \\ &= 100 + 20 \\ &= 120 \end{aligned}$$

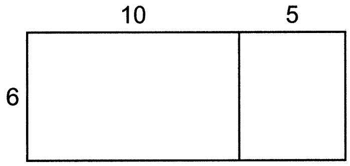
We multiply the term outside the brackets by each term inside the brackets, then find the sum.

This shows the **distributive property** of multiplication.

Check

1. How does each diagram show the distributive property?

a)



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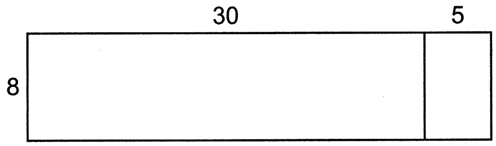
b)



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2. Use the distributive property to multiply.

a) $7 \times 21 = 7 \times (20 + 1)$

$= (7 \times 20) + (7 \times 1)$

= ____________________

= ____________________

b) $8 \times 43 = 8 \times (40 + 3)$

= ____________________

= ____________________

= ____________________

Multiplying and Dividing Integers

When multiplying or dividing 2 integers, look at the sign of each integer:

- When the integers have the same sign, their product or quotient is positive.
- When the integers have different signs, their product or quotient is negative.

×/÷	(−)	(+)
(−)	(+)	(−)
(+)	(−)	(+)

7 × (−4)
7 × (−4) = −28

These 2 integers have different signs, so their product is negative.

(−12) ÷ (−3)
(−12) ÷ (−3) = 4

These 2 integers have the same sign, so their quotient is positive.

When one number is divided by another number, the result is called the quotient.

Check

1. Will the product be positive or negative?

a) 9 × 5 ____________
b) 8 × (−3) ____________
c) (−12) × 5 ____________
d) (−7) × (−6) ____________

2. Multiply.

a) 6 × 5 = ____________
b) 4 × (−10) = ____________
c) (−7) × 3 = ____________
d) (−8) × (−6) = ____________
e) 12 × (−5) = ____________
f) (−4) × (−8) = ____________

3. Will the quotient be positive or negative?

a) 18 ÷ 3 ____________
b) (−36) ÷ 6 ____________
c) 72 ÷ (−9) ____________
d) (−48) ÷ (−8) ____________

4. Divide.

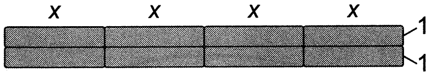
a) (−49) ÷ 7 = ____________
b) (−56) ÷ (−8) = ____________
c) 48 ÷ 6 = ____________
d) 81 ÷ (−9) = ____________
e) (−27) ÷ (−3) = ____________
f) (−42) ÷ 7 = ____________

5.5 Multiplying and Dividing a Polynomial by a Constant

FOCUS **Use different strategies to multiply and divide a polynomial by a constant.**

To multiply $2(4x)$ with algebra tiles:

Model 2 rows of 4 ▭.



There are 8 x-tiles. So, $2(4x) = 8x$

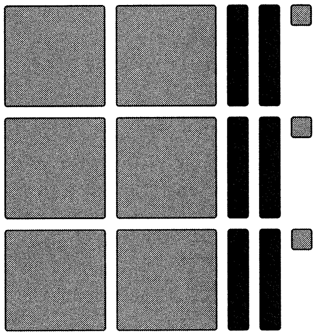
Recall: $2(4x) = 2 \times 4x$

Example 1 Using Algebra Tiles to Multiply a Polynomial by a Constant

Find the product: $3(2b^2 - 2b + 1)$

Solution

$3(2b^2 - 2b + 1)$



Model 3 rows of 2 ▨, 2 ▬, and 1 ▪.
These tiles represent: $6b^2 - 6b + 3$.
So, $3(2b^2 - 2b + 1) = 6b^2 - 6b + 3$

Check

1. Sketch algebra tiles to multiply. Write the product each time.

a) $3(4p - 3) =$ ____________

b) $2(-s^2 + s + 3) =$ ______________

When working symbolically, remember the rules for integer multiplication and division.

Example 2 Using the Distributive Property to Multiply a Polynomial by a Constant

Find the product: $-5(4e^2 - 5e + 3)$

Solution

$-5(4e^2 - 5e + 3)$ Multiply each term in brackets by −5.

$= (-5)(4e^2) + (-5)(-5e) + (-5)(3)$ Multiply.

$= -20e^2 + 25e + (-15)$

$= -20e^2 + 25e - 15$

Check

1. Multiply.

a) $3(7s^2 + 9)$ Multiply each term in brackets by 3.

$= 3(7s^2) + 3(9)$ Multiply: 3 × 7 = ____ and 3 × 9 = ____

$=$ ____s^2 + ____

b) $-4(5e^2 - 8e)$ Multiply each term in brackets by −4.

= ______________________ Multiply.

= ______________________

c) $-5(-2d^2 - 3d + 6)$

= ______________________

= ______________________

= ______________________

d) $7(6y^2 - 8y + 9)$

= ______________________

= ______________________

= ______________________

We can use algebra tiles to divide a polynomial by a constant.

To divide: $(-8x) \div 2$

Arrange 8 ▬ into 2 equal rows.

In each row there are 4 ▬.

So, $(-8x) \div 2 = -4x$

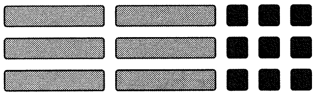
Example 3 Using Algebra Tiles to Divide a Polynomial by a Constant

Find the quotient: $(6s - 9) \div 3$

Solution

$(6s - 9) \div 3$

Arrange 6 ▭ and 9 ■ into 3 equal rows.



In each row, there are 2 ▭ and 3 ■.

So, $(6s - 9) \div 3 = 2s - 3$

Check

1. Sketch algebra tiles to divide. Write the quotient each time.

a) $(3g^2 + 12g) \div 3 =$ ______________ **b)** $(-4b^2 + 6) \div 2 =$ ______________

c) $(4s^2 - 4s + 8) \div 4 =$ ____________ **d)** $(-6t^2 + 9t - 9) \div 3 =$ ____________

When algebra tiles are not available,
or when the divisor is negative,
we can use what we already know about division.

In the division sentence $6 \div 3 = 2$, the divisor is 3.

We can write $8x \div 4$ as a fraction: $\frac{8x}{4}$

We write the fraction as a product, then simplify each fraction.

$$\frac{8x}{4} = \frac{8}{4} \times x$$
$$= 2 \times x$$
$$= 2x$$

Example 4 Dividing a Polynomial by a Constant Symbolically

Find the quotient: $\frac{-9v^2 + 6}{3}$

Solution

$\frac{-9v^2 + 6}{3}$ Write as the sum of 2 fractions with denominator 3.

$= \frac{-9v^2}{3} + \frac{6}{3}$ Simplify the fractions.

$= \frac{-9}{3} \times v^2 + 2$ When 2 integers have different signs, the quotient is negative.

$= -3 \times v^2 + 2$

$= -3v^2 + 2$

Check

1. Divide.

a) $\frac{12r^2 + 8}{4}$ Write as the sum of 2 fractions with denominator 4.

$= \frac{\_\_\_}{4} + \frac{\_\_\_}{4}$ Simplify the fractions.

$= \frac{\_\_\_}{\_\_\_} \times r^2 +$ ________ When 2 integers have the same sign, the quotient is ________.

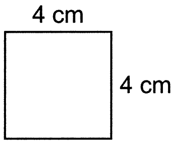
= ____________

= ____________

5.6 Skill Builder

Multiplying Monomials

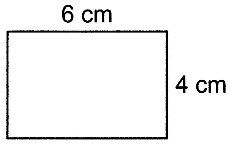
The area of this square is:



$4 \times 4 = 16$

Area $= 16\text{ cm}^2$

The area of this rectangle is:

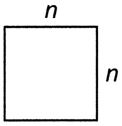


$4 \times 6 = 24$

Area $= 24\text{ cm}^2$

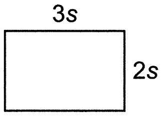
We can use the models above to help us multiply 2 monomials.

The area of this square is:



$n \times n = n^2$

The area of this rectangle is:



$$\begin{aligned} 3s \times 2s &= 3 \times s \times 2 \times s \\ &= 3 \times 2 \times s \times s \\ &= 6 \times s^2 \\ &= 6s^2 \end{aligned}$$

When one or both of the monomials is negative, we cannot use an area model.
We multiply using the rules for multiplying integers.

$$\begin{aligned} 4v \times (-2v) &= 4 \times v \times (-2) \times v \\ &= 4 \times (-2) \times v \times v \\ &= -8 \times v^2 \\ &= -8v^2 \end{aligned}$$

4 and −2 have different signs, so their product is negative.

Check

1. Multiply.

a) $b \times b =$ _____

b) $c \times (-c) =$ _____

c) $(-f) \times (-f) =$ _____

d) $(-g) \times g =$ _____

2. Multiply.

a) $5r \times 6r = 5 \times r \times 6 \times r$
$= 5 \times 6 \times r \times r$
$=$ ____________________
$=$ ____________________

b) $(-2d) \times 8d = (-2) \times d \times 8 \times d$
$= (-2) \times 8 \times d \times d$
$=$ ____________________
$=$ ____________________

c) $4a \times (-7a) =$ ____________________
$=$ ____________________
$=$ ____________________
$=$ ____________________

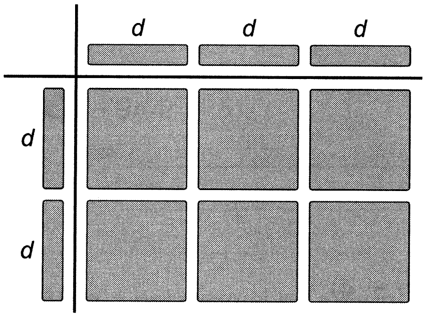
d) $(-5v) \times (-9v) = (-5) \times v \times (-9) \times v$
$=$ ____________________
$=$ ____________________
$=$ ____________________

5.6 Multiplying and Dividing a Polynomial by a Monomial

FOCUS **Use different strategies to multiply and divide a polynomial by a monomial.**

To multiply $2d(3d)$ with algebra tiles:

Draw 2 adjacent sides of a rectangle.
Position ▭ tiles to show side lengths $2d$ and $3d$.



$d \times d = d^2$, so use a d^2-tile.

Then fill the rectangle with tiles.

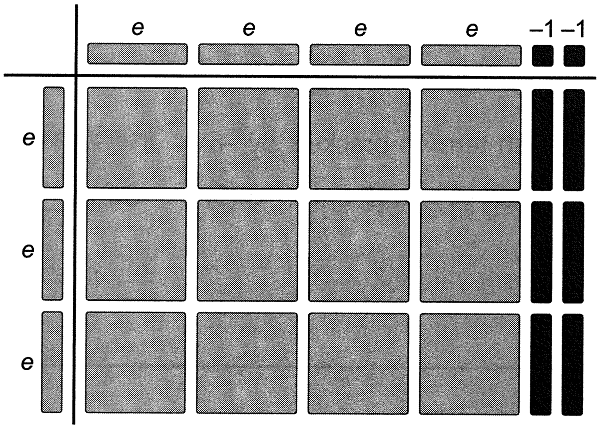
We used 6 d^2-tiles to fill the rectangle. So, $2d(3d) = 6d^2$

Example 1 Multiplying a Binomial by a Monomial

Find the product: $3e(4e - 2)$

Solution

$3e(4e - 2)$
Draw 2 adjacent sides of a rectangle.
Position tiles to show side lengths $3e$ and $4e - 2$.



When two tiles have the same colour, use a positive tile in the rectangle.

When two tiles have different colours, use a negative tile in the rectangle.

Fill the rectangle with tiles.
We used 12 e^2-tiles and 6 $-e$-tiles to fill the rectangle.
So, $3e(4e - 2) = 12e^2 - 6e$

Check

1. Divide.

a) $\frac{12a^2}{-6a}$

Write as a product of 2 fractions.

$= \frac{\_\_\_}{\_\_\_} \times \frac{a^2}{a}$

Simplify each fraction.

$= \_\_\_ \times \frac{a \times \cancel{a}^{1}}{\cancel{a}_{1}}$

= ________

= ________

b) $\frac{9b^2 + 3b}{3b}$

$= \frac{\_\_\_}{3b} + \frac{\_\_\_}{3b}$

= ____________________

= ____________________

= ____________________

c) $\frac{-14c^2 + 21c}{-7c}$

= ____________________

= ____________________

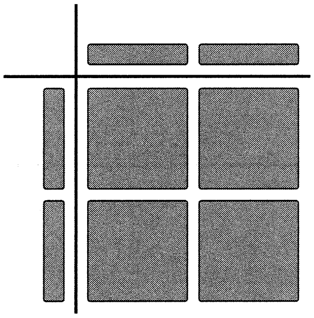
= ____________________

= ____________________

Practice

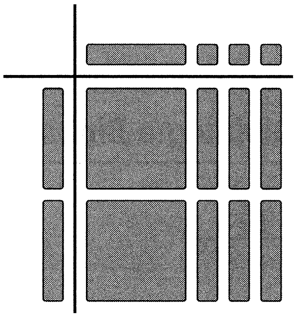
1. Write the multiplication sentence modelled by each set of tiles.

a)



____

b)



____

2. Sketch algebra tiles to multiply. Write the product each time.

a) $2s(s + 4) = \_\_\_\_\_\_\_\_$ **b)** $t(-2t + 3) = \_\_\_\_\_\_\_\_\_\_$

3. Multiply.

a) $4r(5r - 1)$

$= (4r)(\_\_\_) + (4r)(\_\_\_)$

$= \_\_\_r^2 + (\_\_\_r)$

$= \_\_\_\_\_\_\_\_\_\_$

b) $7s(-3s + 6)$

$= \_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$

c) $-6t(t - 3)$

$= \_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$

d) $-8u(-6u + 7)$

$= \_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$

4. Divide.

a) $\frac{12v^2}{4v}$

$= \frac{}{\_\_\_\_} \times \frac{v^2}{v}$

$= \_\_\_\_ \times \frac{v \times \cancel{v}^{1}}{\cancel{v}_{1}}$

$= \_\_\_\_ \times v$

$= \_\_\_\_$

b) $\frac{15w^2}{-3w}$

$= \frac{}{\_\_\_\_\_\_}$

$= \_\_\_\_\_\_$

$= \_\_\_\_\_\_$

c) $\frac{-28x^2}{-7x}$

$= \frac{}{\_\_\_\_\_\_}$

$= \_\_\_\_\_\_$

$= \_\_\_\_\_\_$

5. Divide.

a) $\frac{18y^2 + 12y}{2y}$

$= \frac{\_\_\_}{2y} + \frac{\_\_\_}{2y}$

$= \frac{}{\_\_\_\_\_\_\_\_\_\_}$

$= \_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$

b) $\frac{-32z^2 + 24z}{-8z}$

$= \frac{}{\_\_\_\_\_\_\_\_\_\_}$

$= \frac{}{\_\_\_\_\_\_\_\_\_\_}$

$= \_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$

c) $\frac{15n^2 + 21n}{-3n}$

$= \frac{}{\_\_\_\_\_\_\_\_\_\_}$

$= \frac{}{\_\_\_\_\_\_\_\_\_\_}$

$= \_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$

Unit 5 Puzzle

Alphabet Soup!

The table below contains 15 polynomial expressions.
Simplify each expression.

1 $2x(x - 3)$ ______________	**4** $(3x + 7) - (11 - 4x)$ ______________	**3** $\frac{5x^2 + 10x}{5x}$ ______________
6 $-4(x^2 + 3x - 1)$ ______________	**5** $(5x + 4) + (x^2 - 2x + 1)$ ______________	**7** $3(2x - 1)$ ______________
9 $3x^2 + 5 - 2x$ $- (5 + 3x^2 - 2x)$ ______________	**10** $8 + 7x - 11 - 3x$ ______________	**13** $(4x^2 + 9x + 5)$ $- (4x^2 + 8x + 3)$ ______________
8 $\frac{36x^2 - 18x}{6x}$ ______________	**2** $2x^2 + x + 5 - 7x - 5$ ______________	**11** $(16x - 12) \div 4$ ______________
0 $(6x + 5) + (-9 + x)$ ______________	**12** $2(-2x^2 - 6x + 2)$ ______________	**15** $(4x^2 + x + 6)$ $- (3x^2 - 2x + 1)$ ______________

Seven pairs of expressions have the same answer. Find the 7 pairs.
For each matching pair, add the numbers in the top left corner of each square.
The sum represents a letter of the alphabet.

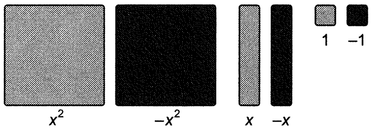
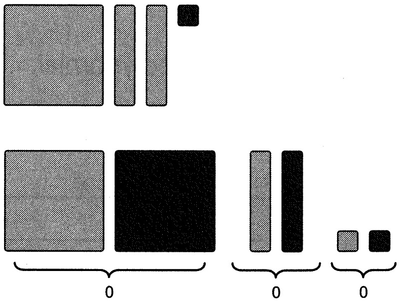
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Write the seven letters below.

___ ___ ___ ___ ___ ___ ___

Unscramble the letters to find a math word used in this unit. ____________

Unit 5 Study Guide

Skill	Description	Example
Recognize the different parts of a polynomial.	A polynomial may have variable terms and a constant term. The number in front of a variable is its coefficient.	variable term $\boxed{3x^2} + 2x + 4$ coefficient constant
Describe and classify polynomials.	A polynomial can be classified by its number of terms and by its term with the greatest degree.	Monomial: $3x$ Binomial: $2x + 5$ Trinomial: $x^2 + 2x - 1$ degree 2
Use algebra tiles to represent a polynomial.	We use these tiles:  A pair of tiles with the same shape and size, but different colours forms a zero pair. The tiles model 0.	$x^2 + 2x - 1$ 
Simplify polynomials by combining like terms.	To simplify a polynomial, add the coefficients of like terms.	Like terms: $4x^2$ and $-2x^2$ Unlike terms: $3x$ and -5 $4x^2 - 2x^2 = 2x^2$
Add polynomials.	To add polynomials, remove the brackets and add the coefficients of like terms.	$(4x^2 + 3x) + (x^2 - 5x)$ $= 4x^2 + 3x + x^2 - 5x$ $= 4x^2 + x^2 + 3x - 5x$ $= 5x^2 - 2x$
Subtract polynomials.	To subtract a polynomial, add the opposite terms.	$(3x^2 + 5x) - (2x^2 - x)$ $= 3x^2 + 5x + (-2x^2 + x)$ $= 3x^2 + 5x - 2x^2 + x$ $= 3x^2 - 2x^2 + 5x + x$ $= x^2 + 6x$
Multiply a polynomial by a monomial.	To multiply a polynomial by a monomial, use the distributive property.	$3x(6x - 5)$ $= 3x(6x) + (3x)(-5)$ $= 18x^2 + (-15x)$ $= 18x^2 - 15x$
Divide a polynomial by a monomial.	To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.	$\frac{24x^2 - 32x}{8x} = \frac{24x^2}{8x} + \frac{-32x}{8x}$ $= 3x - 4$

Unit 5 Review

5.1 **1.** Is the polynomial a monomial, binomial, or trinomial?

a) $-3s^2 + 11$ ____________. **b)** $8d$ ____________.

c) $2e^2 - 9e + 7$ ____________. **d)** $8h - 1$ ____________.

2. Sketch algebra tiles to model each polynomial.

a) $3k - 4$ **b)** $2m^2 - m + 3$ **c)** $-n^2 + 5n - 2$

______________ ______________________ ______________________

5.2 **3.** Simplify each polynomial.

a) $-7d - 4 + 8d + 2$

= ____________________________

= ____________________________

b) $3e^2 - 8e + 2e^2 + 11e$

= ____________________________

= ____________________________

c) $13 - 6h + 2h^2 + 7h - 9$

= ____________________________

= ____________________________

= ____________________________

d) $-9k^2 + 15k - 8 - 2k^2 - 4k + 3$

= ____________________________

= ____________________________

4. Identify and explain any errors you find.

a) $2x^2 + 5x = 7x^2$

____

____

____

b) $5s - 7s = -2s$

____

____

____

5.3 **5.** Sketch algebra tiles to model each sum. Then write the sum.

a) $(-5e + 7) + (4e - 1)$

Remaining tiles: _______________

So, $(-5e + 7) + (4e - 1) =$ __________

b) $(6f^2 - 2f + 5) + (-4f^2 - f - 3)$

Remaining tiles: ________________________

So, $(6f^2 - 2f + 5) + (-4f^2 - f - 3)$

= ____________

6. Add.

a) $(7r + 11) + (-2r + 3)$
$=$ ____________________
$=$ ____________________
$=$ ____________________

b) $(-9s^2 + 5s) + (16s^2 - 9s - 14)$
$=$ ____________________________
$=$ ____________________________
$=$ ____________________________

5.4 **7.** Use algebra tiles to model each difference.
Sketch the tiles that remain, then write the difference.

a) $(-2t + 5) - (-5t + 7)$

Remaining tiles: ______________
So, $(-2t + 5) - (-5t + 7) =$ _____

b) $(-7u - 2) - (-u^2 - 3u - 1)$

Remaining tiles: __________________
So, $(-7u - 2) - (-u^2 - 3u - 1) =$ ___________

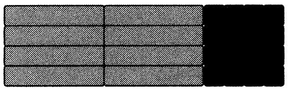
8. Subtract.

a) $(6v + 5) - (13v - 3)$
$= 6v + 5 + ($__________$)$
$=$ ____________________
$=$ ____________________
$=$ ____________________

b) $(10w^2 - 7) - (-2w + 9w^2 + 5)$
$=$ ____________________________
$=$ ____________________________
$=$ ____________________________
$=$ ____________________________

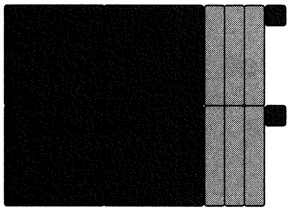
5.5 **9.** Write the multiplication sentence modelled by each set of tiles.

a)



____

b)



____

10. Multiply.

a) $6(-7y^2 + 1)$
$= 6($_____$) + 6($_____$)$
$=$ __________________

b) $-9(-2z^2 - 4z + 5)$
$=$ ___________________________________
$=$ ___________________________________
$=$ ___________________________________

11. Divide.

a) $\frac{16a-40}{8}$

$= \frac{\underline{\qquad}}{8} + \frac{\underline{\qquad}}{8}$

$= \frac{16}{8} \times a + (\underline{\quad})$

$= \underline{\qquad\qquad}$

$= \underline{\qquad\qquad}$

b) $\frac{27b^2 - 9b + 36}{-9}$

$= \underline{\qquad\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad\qquad}$

5.6 **12.** Sketch algebra tiles to multiply. Write the product each time.

a) $2c(c + 5) = \underline{\qquad\qquad}$

b) $3d(-d + 4) = \underline{\qquad\qquad}$

13. Multiply.

a) $3e(5e - 2)$

$= (3e)(\underline{\quad}) + (3e)(\underline{\quad})$

$= \underline{\quad}e^2 + (\underline{\quad})e$

$= \underline{\qquad\qquad}$

b) $-4f(5f + 2)$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

14. Divide.

a) $\frac{-21k^2}{7k}$

$= \frac{-21}{7} \times \frac{k^2}{k}$

$= \underline{\quad} \times \frac{k \times \cancel{k}^{1}}{\cancel{k}_{1}}$

$= \underline{\quad} \times k$

$= \underline{\qquad}$

b) $\frac{81m^2 - 45m}{-9m}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

c) $\frac{-33n^2 + 36n}{-3n}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$